**ME EN 541 – HW 1**

Handed out 12 Jan 2023

Due 20 Jan 2023 at 11:59 pm

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| **Problem** | **Points** | **Topic** |
| **1** | 2 | Gradient operator & dot product practice |
| **2** | 2 | Gradient operator & dot product practice |
| **3** | 4 | Practice simplifying governing equations |
| **4** | 4 | Practice simplifying governing equations |
| **5** | 4 | Practice simplifying governing equations |
| **6** | 8 | Finite difference approximations |
| **7** | 4 | Finite difference approximations |
| **8** | 4 | Finite difference approximations |

1. Show that .

2. Show that the following two forms of the continuity equation are equal:

 & 

3. Starting with the Navier-Stokes equations, ,

obtain an expression for the velocity profile *u*(*y*) for combined pressure-driven and Couette flow (illustrated at right). Give the equation for *u*(*y*) and sketch the velocity profile for the particular case *U* > 0 and *dp*/*dx* < 0.

*x*

*U*

*y*

*dp/dx ≠ 0*

85°C

20°C, *h*

100°C

*x*

*x* = 0

*x =* 1 m

4. A 1-m-long plate (*k* = 50 W/∙mK) is well-insulated on its top and bottom sides, while the left side is at 100°C and the right side is convectively cooled at 20°C. Under steady-state conditions with no generation, a thermocouple at the midpoint of the plate reveals a temperature of 85°C.

1. What is the simplified form of the governing equation?
2. What is the temperature distribution, *T*(*x*)?
3. What is the convection heat transfer coefficient at the right side of the plate?

5. A plane wall (*k* = 50 W/∙mK) is insulated on one side and held at a constant temperature of 0°C on the other. Heat is uniformly generated within the wall at the rate of 1.5×106 W/m3.

0°C

=1.5×106 W/m3

*x*

*x* = 0

*x =* 0.25 m

1. What is the simplified form of the governing equation?
2. What is the temperature distribution, *T*(*x*)?

6. In class we arrived at a general procedure for deriving finite-difference expressions for arbitrary stencils. Use this approach to derive the finite-difference expression of highest order possible for the cases below. Be sure to include the leading truncation error term.

1.  using the stencil (*i* – 1, *i*, *i* + 1)
2.  using the stencil (*i* – 2, *i* – 1, *i*)

7. Use the expressions derived in the previous problem to compute an estimate of the second derivative of ** = sin(*x*) evaluated at *x* = 1.0. Compare the estimates using *x* = 0.2, 0.1, 0.05, and 0.025 with the exact solution. What is the error? How does it change as *x* changes? Is this behavior what you would expect?

8. In class we derived the 2nd-order central finite-difference expression of the governing equation



for constant *k*. Following a similar procedure, derive the expression for the case where *k* = *k*(*x*). In doing so, assume *k* varies linearly between node points.